of smoking is considerably diminished; likewise the successful application of naphthalized gas, and of an oil-lamp, to photographic registration.

The paper concludes with the description of a new method of determining the scale and temperature coefficients of the force magnetometers, by which a greater degree of accuracy is presumed to be attained than by the methods ordinarily employed. Two magnets, designed for self-registering instruments for the observatories at Cambridge and Toronto, having been submitted to this method, gave consistent results which indicate the law of the temperature coefficient to be sensibly different from that which has hitherto been assumed.

12. "On certain Properties of the Arithmetical Series whose ultimate differences are constant." By Sir Frederick Pollock, Lord Chief-Baron of the Exchequer, F.R.S. &c.

This paper professes to investigate certain properties of the series of whole numbers whose ultimate differences are constant, and incidentally to treat of Fermat's theorem of the polygonal numbers, and some other properties of numbers.

Its object is to show that the same (or an analogous) property which Fermat discovered in the polygonal numbers belongs to other series of the same order, also to all series of the first order, and probably to all series of all orders. It also proposes to prove the first case of Fermat's theorem (that is of the triangular numbers) from the second case of the squares (which had not before been done), and to dispense with the elaborate proof of Legendre (Théorie des Nombres), finally, to prove all the cases by a method different from that either of Lagrange, Euler, or Legendre.

It is first shown that an analogous property belongs to all series of the first order (viz. common arithmetical series). The following propositions are then proved as the basis of future reasoning:—

1. Every triangular number greater than 6 is composed of 3 triangular numbers.
2. Every triangular number greater than 3 is composed of 4 triangular numbers.
3. Any triangular number may be expressed by the form $(a^2 \pm a + b^2)$.
4. The sum of any two triangular numbers may be expressed by the same form.
5. Every number above 7 is the sum of *four* triangular numbers *exactly*.
6. Every number above 29 is the sum of *three* triangular numbers *exactly*.
7. Every multiple of 8 is composed of eight odd squares, and the sum of any 8 odd squares is a multiple of 8.
8. The following general theorem is then proved:—

If p be any odd square, then

$$\mathrm{A}p^z + \mathrm{B}p^y + \mathrm{C}p^x + \mathrm{D}p^w + \ \&c.$$

will equal 8 odd squares, if

$$\mathrm{A} + \mathrm{B} + \mathrm{C} + \mathrm{D} +, \ \&c.$$

equal 8, or any multiple of 8 (A, B, C, D, &c., z, y, x, &c, may be any positive whole numbers).

9. It is a corollary from this, that in any system of notation having an odd square for its base, the sum of the digits will have the same number of odd squares as the number itself; the number of odd squares being in each case the minimum.

10. Any number of the form $8n+4$ is composed of 4 odd squares.

11. It follows from this that every number is composed of 4 triangular numbers, or 3, 2 or 1.

12. From this it is shown that every number is of the form

$$(a^2 \pm a + b^2,\ c^2 \pm c + d^2).$$

13. And that every number of the form $4n+2$ (if n be greater than 2) is composed of 4 square numbers, 2 even and 2 odd.

14. And that every number greater than 27 is composed of 8 squares exactly.

15. Every number (beyond a certain small limit) is composed of 2 triangular numbers + a square number, or 2 triangular numbers + a double triangular number.

16. A proof is then offered that in the equation $8n+4=4$ odd squares, one of the four odd squares may be any odd square less than $8n+4$, and therefore 1 may be one of the 4 odd squares; and if so,

$$8n+4=1+3 \text{ odd squares},$$

$$\therefore 8n+3=3 \text{ odd squares},$$

from which Fermat's theorem of the triangular numbers is an immediate corollary.

17. A proof (by a tabular series) is then suggested, that all the other cases of Fermat's theorem may be deduced from the first, and that it is not necessary to use more than four terms greater than unity (as discovered by M. Cauchy, see Suppl. to Legendre's Théorie des Nombres, p. 21, 22).

18. A general expression for a succession of series is then given—

$$1, (p+1), (3p+1), (6p+1) \text{ \&c. } \left(\frac{\overline{n-1}.n}{2}p+1\right);$$

and it is proved that any number may be composed of not exceeding $p+2$ terms of the series, of which three only are required to be greater than unity. If $p=9$, the series is 1, 10, 28, 55, &c., that is every third triangular number beginning with 1; and every number of the form $9n+q$ consists of q triangular numbers not divisible by 3 (if q be greater than 2).

If $p=8$, the series is 1, 9, 25, 49, &c.

19. (The odd squares) *and every number may be composed, of not exceeding* 10 *odd squares.* If $p=6$, the series becomes 1, 7, 19, 37 &c. (the differences between the cubes). From the continued addition of the terms of this series, the cube numbers may be formed.

If $p=4$, the series is 1, 5, 13, 25 &c., or

$$1, (1+4), (4+9), (9+16), \text{ \&c.}$$

The continued addition of the terms of this series forms the octohedral numbers, viz. 1, 6, 19, 44, &c.

Every term of the series is composed of $(p+1)$ prior terms ; also if q be added to any term it will equal $(q+1)$ prior terms.

20. It is then proved that the property of the triangular numbers is not destroyed by adding any (the same) number to each term, it is merely postponed, and commences at a higher number according to the magnitude of the number added.

21. The same is proved in respect of the addition of any common arithmetical series.

22. The paper concludes by suggesting a proof that every number may be composed of 4 triangular numbers, derived from the consideration that if the triangular numbers be indexed or numbered thus—

1	2	3	4	5 &c.	indices.
1	3	6	10	15 &c.	Δ^n nos.

Any number between 2 triangular numbers can be formed by 4 triangular numbers, the sum of whose indices shall be not less than the sum of the indices of the 4 triangular numbers that compose the smaller triangular number, and not greater than the similar indices of the larger; and generally (after a limited number of terms) the sum of the indices of any intermediate number will be exactly the sum of the indices of the smaller number: to illustrate this, all the numbers between 91 and 105 (2 triangular numbers) are shown to consist of 4 triangular numbers, whose indices exactly equal 25, which is the sum of the indices of the 4 triangular numbers into which 91 may be divided—

$$\text{thus } 91=\overset{6}{21}+\overset{6}{21}+\overset{6}{21}+\overset{7}{28}$$

the sum of the indices $6+6+6+7=25$; and every number between 91 and 105 may be composed of 4 triangular numbers, whose indices added together will equal 25; but the nature of this investigation cannot be made intelligible without reference to the table itself, which the paper contains.

If this attempt is successful, the whole of Fermat's theorem of the polygonal numbers may be proved without reference to Lagrange's proof of the case of the squares (the second case) derived from the properties of the prime numbers. The writer intimates an intention of making further communications on the same subject.

13. "On the Analysis of Numerical Equations." By J. R. Young, Esq., Professor of Mathematics in Belfast College. Communicated by Sir John W. Lubbock, Bart., F.R.S. &c.

The object of this communication is to diminish the labour attendant upon existing methods for the analysis of numerical equations. As Budan pointed out intervals, within the bounds of the extreme limits of the roots of an equation, in which all search for roots would be fruitless, so here the author seeks for what he terms "*rejective intervals*" among those which Budan had retained. This he proposes effecting by transforming the first member of every equation $X=0$ into

$$X=\{F+\sqrt{F^2-X}\}\times\{F-\sqrt{F^2-X}\} \quad . \; . \; . \; . \; . \quad (1.)$$

which the author calls decomposing it into *conjugate factors*; in